EEEN 464 – DIGITAL COMMUNICATION

INTER-SYMBOL INTERFERENCE - STUDY GUIDE/REVISION

---

1. FUNDAMENTAL CONCEPTS

1.1 Definition

Inter-Symbol Interference (ISI) occurs when consecutive symbols in a digital transmission system interfere with each other, causing signal distortion and decision errors at the receiver.

This distortion is primarily caused by limitations in the channel's bandwidth and can lead to errors in data transmission.

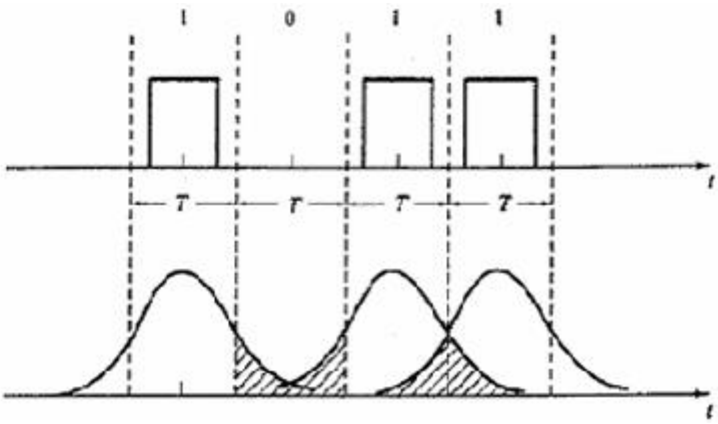

Figure 1. Inter-symbol interference.

1.1 Primary Causes

- **Bandlimited Channels**: Frequency constraints cause pulse spreading beyond symbol duration
- **Multipath Propagation**: Signal reflections create delayed replicas (common in wireless)
- **Nonlinear Phase Response**: Frequency-dependent delays distort pulse shape

1.3 Effects of Inter-symbol Interference

- **Increased Error Probability:** ISI causes the receiver to misinterpret the received signal, leading to errors in decoding the transmitted data.
- **Degraded Signal Quality:** The overlapping symbols distort the signal, making it harder for the receiver to distinguish between different signal levels.
- **Reduced Data Rate:** To mitigate ISI, systems might need to reduce the data transmission rate or employ techniques like equalization, which can limit the overall throughput.
- **Channel Capacity Reduction:** ISI can significantly impact the capacity of a communication channel, limiting the amount of data that can be reliably transmitted.

- **Complexity in Equalization:** ISI often requires the use of equalization techniques at the receiver to compensate for the channel's distorting effects, adding complexity to the receiver design

2. MATHEMATICAL REPRESENTATION

Received Signal Model:

$$r(t)=\sum_n a_n \cdot h(t-nT_s)+n(t)$$

Where:

- a_n = Transmitted symbols
- $h(t)$ = Channel impulse response
- T_s = Symbol period
- $n(t)$ = Additive noise

Sampled Signal at $t=kT_s$:

$$r(kT_s)=a_k h(0)+\underbrace{\sum_{n\neq k} a_n h((k-n)T_s)}_{\text{ISI}}+n(kT_s)$$

Critical Parameter:

- **Delay Spread (τ_{max})**: Maximum excess delay in multipath channels
- ISI becomes severe when $\tau_{max}>0.1T_s$

3. NYQUIST CRITERIA FOR ZERO ISI

First Criterion (Time Domain):

$$h(kT_s)=\begin{cases}1 & k=0\\ 0 & k\neq 0\end{cases}$$

Interpretation: Impulse response has zero crossings at all symbol instants except current.

Second Criterion (Frequency Domain):

$$\sum_{m=-\infty}^{\infty} H\left(f-\frac{m}{T_s}\right)=T_s$$

Interpretation: Folded frequency response must be constant ("vestigial symmetry").

Ideal Solution: Sinc pulse $h(t)=\text{sinc}(t/T_s)$

- **Limitation**: Infinite duration, unrealizable, sensitive to timing errors

4. PULSE SHAPING TECHNIQUES

Raised Cosine Filters:

$$h(t)=\text{sinc}\left(\frac{t}{T_s}\right)\cdot\frac{\cos(\pi\alpha t/T_s)}{1-(2\alpha t/T_s)^2}$$

- **Roll-off Factor (α)**:
  - $\alpha=0$: Ideal sinc (brickwall frequency response)
  - $\alpha=1$: Smoother roll-off (50% excess bandwidth)

- **Trade-off**: Higher α → More bandwidth, less timing sensitivity

Square-Root Raised Cosine (SRRC):

- Used in transmitter and receiver for matched filtering
- Ensures zero ISI at sampling instants

5. EYE DIAGRAMS: VISUAL ISI ANALYSIS

Construction Method:

1. Overlap oscilloscope traces of signal segments (2-3 symbol periods)
2. Synchronize triggering to symbol rate

Key Parameters:

- **Eye Opening**: Vertical distance between rails (indicates noise margin)
- **Eye Width**: Horizontal opening (indicates timing jitter tolerance)
- **Closure**: Indicates ISI severity

Interpretation:

<div style="text-align:center;margin:15px 0"> <pre> Wide Open Eye → Low ISI/Nice Eye Diagram |‾‾‾‾‾‾| Closed Eye → High ISI | | /‾‾‾‾\ |______| / \ </pre> </div>

6. EQUALIZATION TECHNIQUES

Linear Equalizers:

- **Zero Forcing (ZF)**:

$C(z)=\frac{1}{H(z)}$

Pro: Eliminates ISI completely
Con: Amplifies noise at spectral nulls

- **MMSE**: Minimizes $E[|a_k-\hat{a}_k|^2]$
  Better noise-ISI trade-off than ZF

Decision Feedback Equalizer (DFE):

plaintext

Copy

Download

```
       +---------+     +---------+
r(t) --->| Forward |----->| Detector |---> Output
       | Filter |     +---------+
       +---------+          |
```

```
^          |
| +---------+ |
+--| Feedback |<+
  | Filter  |
  +---------+
```

- *Forward filter*: Mitigates precursor ISI
- *Feedback filter*: Cancels postcursor ISI using previous decisions

Adaptive Algorithms:

- LMS: $w_{k+1}=w_k+\mu\cdot e_k\cdot r_k$
  (μ = step size, e_k = error)
- RLS: Faster convergence but higher complexity

7. ADVANCED MITIGATION TECHNIQUES

Orthogonal Frequency Division Multiplexing (OFDM):

- Divides channel into narrow subcarriers
- Converts frequency-selective fading → flat fading
- **Cyclic Prefix**: Absorbs multipath delay spread

$T_{CP}>\tau_{max}$

Maximum Likelihood Sequence Estimation (MLSE):

- Viterbi algorithm implementation
- Optimal performance but computationally intensive ($O(M^L)$ complexity)

8. SYSTEM DESIGN CONSIDERATIONS

Symbol Rate Selection:

- Higher data rates → Shorter T_s → More ISI sensitivity
- Rule of thumb: $T_s>10\cdot\tau_{max}$

Modulation Impact:

- Higher-order QAM more sensitive to ISI than PSK

Channel Estimation:

- Use training sequences (preamble)
- Periodic pilot symbols

9. PRACTICAL EXAMPLES

Wireless Communications:

- GSM: Uses 26-bit training sequence for adaptive equalization
- 5G NR: OFDM with configurable cyclic prefix

Wired Communications:

- Gigabit Ethernet: Tomlinson-Harashima precoding
- Optical Fiber: Coherent detection with digital equalization

10. KEY FORMULAS & RELATIONSHIPS

1. **Excess Bandwidth**:

$\mathrm{Bex{=}\alpha 2Ts}Bex=2Ts\alpha$

2. **RMS Delay Spread**:

$\mathrm{\tau RMS{=}\sum\tau i2P(\tau i)\sum P(\tau i){-}(\tau avg)2}\tau RMS=\sum P(\tau i)\sum\tau i2P(\tau i)-(\tau avg)2$

3. **Coherence Bandwidth**:

$\mathrm{Bc{\approx}15\tau RMS}Bc\approx 5\tau RMS1$

4. **ZF Equalizer Tap Weights**:

$\mathrm{wZF{=}(HTH){-}1HTe}\mathbf{w}ZF=(\mathbf{H}T\mathbf{H})-1\mathbf{H}T\mathbf{e}$

Where H**H** is channel convolution matrix